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DETERMINATION OF THE HEAT TRANSFER COEFFICIENTS IN POROUS MEDIA

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The process of transpiration cooling is considered. Methods are suggested for estimating the volumetric coefficient of heat transfer with the use of a two-temperature model and the surface heat transfer coefficient at entry into a porous wall.

The development of new technology under conditions of increasing heat loads puts the search for effective methods of heat transfer enhancement in the forefront of theoretical investigations. One of the promising trends in the solution of this problem is the use of porous materials (PM) in the elements of power units. For thermal protection against convective or radiative heat fluxes, the method of transpiration cooling is successfully used. The mechanism operative in the thermal protection involves the injection of a coolant through a porous medium to produce a screen over the contour of a body in a flow for removing heat energy from the skeleton of the porous material. Usually, the mathematical simulation of thermal interaction between a gas and a PM skeleton is based on the notion of the volumetric heat transfer coefficient α_V, whose value is assumed to be known a priori. However, examination of a number of experimental and theoretical works on this basic characteristic of heat transfer in PM [1-5] has revealed a discrepancy between them, thus preventing the formulation of a generalizing inference as to the possibility of a unified approach to its determination. At the same time, as regards the degree of universality of a methodological approach, a technique based on the solution of inverse heat conduction problems (IHCP) is shown to be superior [4]. In such a case, the problem of the preliminary estimation of the coefficient sought is of some practical and theoretical interest. As noted in [1, 2], the transfer of heat from the surface of pores to the medium injected is characterized by the surface heat transfer coefficient α_S. The two coefficients are interrelated by

$$\alpha_V = \alpha_S \frac{S}{V}, \tag{1}$$

where S is the active surface of the pores. Nevertheless, relation (1) has not been utilized extensively because of the great difficulties posed by the determination of the active surface of pores in real PM, the discrete character of the medium, different shapes of elementary particles, and the impossibility of measuring exactly the temperature of the skeleton and gas in the pores. Many complexities also arise in the formulation of the boundary conditions at entry into a PM, since there are virtually no theoretical relations for calculating the surface heat transfer coefficient α_w [2].

In view of this, in the present paper homogeneous porous materials in the form of a cylindrical region with structure elements representing spherical and cylindrical particles, a fiber, and a strand of fibers are investigated. It is assumed that elementary particles are in point contact and that the cooling gas injection and thermal loading are implemented from the ends of the isolated region. Then, taking into account the paucity of blind and closed pores compared to the fraction of transporting pores, it is possible to calculate analytically the area of the active pore surface of porous media with different elementary particles; the specific expressions are listed in Table 1. For the same volumetric fraction of the skeleton V_{sk} we obtain from this table a simple relation expressing the equivalence of the active surfaces

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TABLE 1. Transition from a Porous Material to a Perforated Region

Element of porous material structure	Active surface of pores S, m^2	Dependence of channel radius R_{ch}, m, on size of particles	Volumetric Nusselt number and number of channels in equivalent region
Spherical particle	$S = \frac{3V_{sk}}{R_{sph}}$	$R_{ch} = \frac{2}{3}\varphi R_{sph}$	
Cylindrical particle	$S = \frac{2(L+R)}{LR}$	$R_{ch} = \varphi \frac{LR}{L+R}$	$Nu_V = 2\Pi\, Nu_S$
Fiber	$S = \frac{2V_{sk}}{R_f}$	$R_{ch} = \varphi R_f$	$n = \frac{1}{\Pi}\left(\frac{S}{F}\right)^2$
Strand of fibers	$S = \frac{2V_{sk}}{\left(\frac{N-m}{2}+1\right)r}$	$R_{ch} = \varphi\left(\frac{N-m}{2}+1\right)r$	

$$\frac{3}{R_{sph}} = \frac{2(L+R)}{LR} = \frac{2}{R_f} = \frac{2}{\left(\frac{N-m}{2}+1\right)r}.$$

In the case of a strand of N fibers, it was noted that the internal fibers, m in number in a plait, which do not participate in convective heat transfer, do not bear a useful load in formula (1). Therefore, as the characteristic dimension of a strand of fibers the radius of an effective single fiber with surface equal to the active area of the external fibers in the plait was taken.

Usually, real PMs are investigated with the help of specific physical models, for example, a system of differently packed small-radius spherical particles or an array of communicating cylindrical capillaries. The simplest model of a PM, which represents a bundle of straight cylindrical channels, is considered. In contrast to [1], where such a model transition from a PM to a perforated region with cylindrical pores is made, it is suggested to impose an additional condition requiring the same active surface for the model and real pores. This results in a more regorous account of the individuality and specificity of the initial material. In Table 1 this transition is expressed not only as the dependence of the radius of the cylindrical channel on the size of elementary particles, but also as the formula for calculating the number of capillaries n needed to determine the density of perforation. Moreover, to describe the thermal process in an equivalent model medium on the basis of relation (1), it is possible to construct a dimensionless equation that relates the volumetric Nusselt number to the material porosity and the surface number Nu_S

$$Nu_V = 2\Pi\, Nu_S, \tag{2}$$

where the channel radius is taken to be the characteristic dimension. It should be noted that for a number of cases the formula obtained makes it possible to determine the intensity of volumetric heat transfer in a PM without resorting to the additional specification of Nu_S, since in the literature there are data on heat transfer in tubes and channels both at entry and in the downstream region. Moreover, relation (2) not only is consistent with relation (1), but even augments it with an important parameter - the porosity Π, as confirmed by the conclusion from an experimental investigation [6] of a substantial effect of porosity on α_V.

Performing the transition from a PM with a structure element in the form of a spherical particle with the parameters $D = 5\cdot10^{-2}$ m, $H = 4\cdot10^{-3}$ m, $d = 6.3\cdot10^{-4}$ m, $\Pi = 0.3455$, and $M = 25.65\cdot10^{-3}$ kg/sec [4] to a model perforated region with cylindrical capillaries with the help of the corresponding formulas from Table 1, we obtain: $R_{ch} = 1.109\cdot10^{-4}$ m, $S = 4.893\cdot10^{-2}$ m^2, $V = 7.854\cdot10^{-6}$ m^3, $F = 6.283\cdot10^{-4}$ m^2, $n = 17{,}554$. Moreover, in the

indicated work in three versions of temperature measurements in a PM the following values of the volumetric heat transfer coefficient were reconstructed from the solution of the inverse heat conduction problem: $\alpha_V = 2.668 \cdot 10^6$; $7.251 \cdot 10^5$; $1.452 \cdot 10^6$ W/(m^3·K). At the same time, the calculation for the model perforated region using dimensionless equation (2) with known Nusselt numbers on stabilized segments of heat transfer and hydrodynamics in tubes at a constant temperature T_w or heat flux q_w on the wall [7] respectively yields

$$\text{at} \quad T_w = \text{const} \quad \text{Nu}_S = 3.568, \quad \alpha_V = 2.75 \cdot 10^6;$$

$$\text{at} \quad q_w = \text{const} \quad \text{Nu}_S = 4.365, \quad \alpha_V = 3.28 \cdot 10^6.$$

As is seen from comparison of α_V data obtained by solving the inverse problem and calculated from relation (2), the best agreement is observed when the wall temperature is constant, which corresponds to the conditions of the experiment in [4]. Note that the above example reflects the existence of a lower limit for the applicability of formula (2) because of the use of asymptotic values of Nu_S number downstream. However, in the general case of unsteady-state convective heat transfer in a porous medium the internal heat transfer should be investigated by proceeding from the determination of Nu_S on the initial thermohydrodynamic segments of a model capillary.

The mathematical simulation of transpiration cooling also presupposes account for the specific properties in the statement of boundary conditions. However, the complex character of heat transfer at entry into a porous wall is reflected in the diversity of conditions suggested. The most substantiated of them were proposed by the authors of [2], who distinguished two groups of conditions in accordance with the arrangement of injection. With injection along the normal to the porous material surface

$$\lambda \frac{dT}{dx} = Gc\,(t - t_0); \tag{3}$$

$$\lambda \frac{dT}{dx} = \alpha_w\,(T - t_0) \quad \text{or} \quad \frac{\alpha_w}{Gc} = \frac{t - t_0}{T - t_0} = \text{St}_w, \tag{4}$$

and with flow along the porous wall

$$\lambda \frac{dT}{dx} = \alpha\,(T - t_0); \tag{5}$$

$$\lambda \frac{dT}{dx} = Gc\,(t - t_0) \quad \text{or} \quad \frac{\alpha}{Gc} = \text{St}_w, \tag{6}$$

where α is the effective coefficient of heat transfer from a permeable surface to the coolant entering into it; St_w is the Stanton number of heat transfer at entry into a porous wall, characterizing the relative heating of a gas. At the same time it is noted that the parameters α_w and α are not determined theoretically and that few experimental investigations for entry into a porous wall are generalized as the dimensionless relation $\text{Nu}_w = \text{Pe}^n$, $n \leq 1$.

Consideration of the structure of a PM and its analysis allows one in each specific case to isolate a characteristic element of the medium with its inherent process of heat transfer and hydrodynamics, whose repetition reproduces not only the structure of the PM, but the entire pattern of heat transfer and flow at entry into a porous wall. Such elements are taken to be a spherical particle and a cylindrical fiber, as being the most characteristic. At the same time, the results of theoretical investigations based on the solutions of equations in the boundary layer approximation are used, which are presented in the form of dimensionless heat transfer relations for flows past a sphere and a cylindrical tube. The maximum Nusselt numbers are observed at the stagnation points [7, 8]:

$$\text{Nu} = 1.315\,\text{Re}^{0.5}\,\text{Pr}^{0.4} \quad \text{for a sphere}; \tag{7}$$

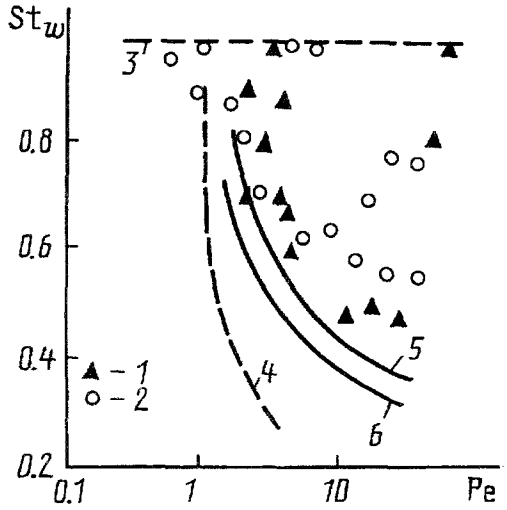

Fig. 1. Comparison of theoretical and experimental results for St_w; 1, 2) data of [2] for $d = 100$ μm and $d = 350$ μm; 3) $St_w = 1$; 4) $St_w = Pe^{-0.765}$; 5, 6) formula (9) for $A = 1.315$ and 1.14, respectively.

$$Nu = 1.14\,Re^{0.5}\,Pr^{0.4} \quad \text{for a cylinder}, \tag{8}$$

where the diameter of the sphere and cylinder is taken as the characteristic dimension. The transformation of Eqs. (7) and (8) to be incorporated in Eqs. (4) and (6) gives the expression

$$St_w = \frac{A}{Re^{0.5}\,Pr^{0.6}}, \tag{9}$$

where $A = 1.315$ in the case of spherical particles and $A = 1.14$ for cylindrical particles. The validity of such a transition from a unit element to an ensemble of particles on the surface of a PM is based first of all on the fact of satisfactory agreement between relation (9) and experimental investigations at entry into a porous wall borrowed from [2] (see Fig. 1). Curves 5 and 6, constructed by formula (9), reflect the change in St_w for porous media made from spherical particles and fibers, respectively. In [4], in addition to the reconstruction of the volumetric heat transfer coefficient, the surface coefficient of heat transfer at entry into a porous wall was determined in the form $\alpha_w = 350$ W/(m^2·K). Calculation by relation (9) for numerical comparison using the data for air $c = 1005.7$ J/(kg·K), $\mu = 1.983 \cdot 10^{-5}$ kg/(m·sec), $Pr = 0.708$, $T_0 = 300$ K gave the close value $\alpha_w = 353.3$ W/(m^2·K). A similar comparative analysis with [9], where the heat transfer coefficient at entry was determined to be $\alpha_w = 1160$, revealed satisfactory agreement with $\alpha_w = 1043$ according to relation (9). The condition $St_w \leq 1$ for the relative heating of the gas at entry into a PM to be limited, which follows from Eqs. (4) and (6), is also valid for Eq. (9).

Thus, the physical consistency and agreement with experimental data allow us to recommend the use of the techniques of estimating the volume and surface heat transfer coefficients in the design of porous materials and the evaluation of their heat exchange properties. The results can be employed for improving the production technology of porous materials and as a priori information for solving IHCP.

NOTATION

R_{sph}, R and L, R_f, r, characteristic dimensions of particles in the form of a sphere, cylinder, single fiber, and plaited fiber; Nu_V, volumetric Nusselt number; λ, thermal conductivity coefficient of the coolant; Nu_w, Nusselt number at entry into a porous wall; Pe, Re, Pr, Peclet, Reynolds, and Prandtl numbers; μ, dynamic coefficient of viscosity; T, T_0, temperature of the skeleton and initial gas temperature; d, particle diameter; D, H, F, V, diameter, thickness, side surface, and volume of a sample; M, mass flow rate of the gas; $\varphi = \Pi/(1-\Pi)$; G, specific mass flow

rate of the gas; c, specific heat capacity of the gas; t, t_0, gas temperature at entry and before entry into a porous material.

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